PART I SCHWESER'S QuickSheet™

CRITICAL CONCEPTS FOR THE 2016 FRM® EXAM

FOUNDATIONS OF RISK MANAGEMENT

Types of Risk

Key classes of risk include market risk, credit risk, liquidity risk, operational risk, legal and regulatory risk, business risk, strategic risk, and reputation risk.

- *Market risk* includes interest rate risk, equity price risk, foreign exchange risk, and commodity price risk.
- *Credit risk* includes default risk, bankruptcy risk, downgrade risk, and settlement risk.
- *Liquidity risk* includes funding liquidity risk and trading liquidity risk.

Enterprise Risk Management (ERM)

Comprehensive and integrated framework for managing firm risks in order to meet business objectives, minimize unexpected earnings volatility, and maximize firm value. Benefits include (1) increased organizational effectiveness, (2) better risk reporting, and (3) improved business performance.

Determining Optimal Risk Exposure

Target certain default probability or specific credit rating: high credit rating may have opportunity costs (e.g., forego risky/profitable projects).

Sensitivity or scenario analysis: examine adverse impacts on value from specific shocks.

Diversifiable and Systematic Risk

The part of the volatility of a single security's returns that is uncorrelated with the volatility of the market portfolio is that security's **diversifiable risk**.

The part of an individual security's risk that arises because of the positive covariance of that security's returns with overall market returns is called its systematic **risk**.

A standardized measure of systematic risk is **beta**:

$$\text{beta}_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

Capital Asset Pricing Model (CAPM)

In equilibrium, all investors hold a portfolio of risky assets that has the same weights as the market portfolio. The CAPM is expressed in the equation of the security market line (SML). For any single security or portfolio of securities *i*, the expected return in equilibrium, is:

$$E(R_i) = R_F + \text{beta}_i[E(R_M) - R_F]$$

CAPM Assumptions

- Investors seek to maximize the expected utility of their wealth at the end of the period, and all investors have the same investment horizon.
- Investors are risk averse.
- Investors only consider the mean and standard deviation of returns (which implicitly assumes the asset returns are normally distributed).
- Investors can borrow and lend at the same risk-free rate.
- Investors have the same expectations concerning returns.
- There are neither taxes nor transactions costs, and assets are infinitely divisible. This is often referred to as "perfect markets."

Arbitrage Pricing Theory (APT)

The APT describes expected returns as a linear function of exposures to common risk factors:

$$E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \ldots + \beta_{ik}RP_k$$

where:

β_{ij} = j^{th} factor beta for stock i

RP_j = risk premium associated with risk factor j

The APT defines the structure of returns but does not define which factors should be used in the model.

The *CAPM* is a special case of APT with only one factor exposure—the market risk premium.

The *Fama-French three-factor model* describes returns as a linear function of the market index return, firm size, and book-to-market factors.

Measures of Performance

The **Treynor** measure is equal to the risk premium divided by beta, or systematic risk:

$$\text{Treynor measure} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

The **Sharpe** measure is equal to the risk premium divided by the standard deviation, or total risk:

$$\text{Sharpe measure} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

The Jensen measure (a.k.a. Jensen's alpha or just alpha), is the asset's excess return over the return predicted by the CAPM:

$$\text{Jensen measure} = \alpha_P = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

The **information ratio** is essentially the alpha of the managed portfolio relative to its benchmark divided by the tracking error.

$$IR = \left[\frac{E(R_P) - E(R_B)}{\text{tracking error}}\right]$$

The **Sortino ratio** is similar to the Sharpe ratio except we replace the risk-free rate with a minimum acceptable return, denoted R_{min}, and we replace the standard deviation with a type of semi-standard deviation.

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\text{semi-standard deviation}}$$

Financial Disasters

Drysdale Securities: borrowed $300 million in unsecured funds from Chase Manhattan by exploiting a flaw in the system for computing the value of collateral.

Kidder Peabody: Joseph Jett reported substantial artificial profits; after the fake profits were detected, $350 million in previously reported gains had to be reversed.

Barings: rogue trader, Nick Leeson, took speculative derivative positions (Nikkei 225 futures) in an attempt to cover trading losses; Leeson had dual responsibilities of trading and supervising settlement operations, allowing him to hide trading losses; lessons include separation of duties and management oversight.

Allied Irish Bank: currency trader, John Rusnak, hid $691 million in losses; Rusnak bullied back-office workers into not following-up on trade confirmations for fake trades.

UBS: equity derivatives business lost millions due to incorrect modeling of long-dated options and its stake in Long-Term Capital Management.

Société Générale: junior trader, Jérôme Kerviel, participated in unauthorized trading activity and hid activity with fake offsetting transactions; fraud resulted in losses of $7.1 billion.

Metallgesellschaft: short-term futures contracts used to hedge long-term exposure in the petroleum markets; stack-and-roll hedging strategy; marking to market on futures caused huge cash flow problems.

Long-Term Capital Management: hedge fund that used relative value strategies with enormous amounts of leverage; when Russia defaulted on its debt in 1998, the increase in yield spreads caused huge losses and enormous cash flow problems from realizing marking to market losses; lessons include lack of diversification, model risk, leverage, and funding and trading liquidity risks.

Banker's Trust: developed derivative structures that were intentionally complex; in taped phone conversations, staff bragged about how badly they fooled clients.

JPMorgan and Citigroup: main counterparties in Enron's derivatives transactions; agreed to pay a $286 million fine for assisting with fraud against Enron investors.

Role of Risk Management

1. Assess all risks faced by the firm.
2. Communicate these risks to risk-taking decision makers.
3. Monitor and manage these risks.

Objective of risk management is to recognize that large losses are possible and to develop contingency plans that deal with such losses if they should occur.

Risk Data Aggregation

Defining, gathering, and processing risk data for measuring performance against risk tolerance.

Benefits of effective risk data aggregation and reporting systems:

- Increases ability to anticipate problems.
- Identifies routes to financial health.
- Improves resolvability in event of bank stress.
- Increases efficiency, reduces chance of loss, and increases profitability.

GARP Code of Conduct

Sets forth principles related to ethical behavior within the risk management profession.

It stresses ethical behavior in the following areas:

Principles

- Professional integrity and ethical conduct
- Conflicts of interest
- Confidentiality

Professional Standards

- Fundamental responsibilities
- Adherence to best practices

Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership. In addition, violations could lead to a revocation of the right to use the FRM designation.

QUANTITATIVE ANALYSIS

Probabilities

Unconditional probability (marginal probability) is the probability of an event occurring.

Conditional probability, P(A | B), is the probability of an event A occurring given that event B has occurred.

Bayes' Theorem

Updates the prior probability for an event in response to the arrival of new information.

$$P(I \mid O) = \frac{P(O \mid I)}{P(O)} \times P(I)$$

Expected Value

Weighted average of the possible outcomes of a random variable, where the weights are the probabilities that the outcomes will occur.

$$E(X) = \sum P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$$

Variance

Provides a measure of the extent of the dispersion in the values of the random variable around the mean. The square root of the variance is called the standard deviation.

$$\text{variance}(X) = E[(X - \mu)^2]$$

Covariance

Expected value of the product of the deviations of two random variables from their respective expected values.

$$Cov(R_i, R_j) = E\{[R_i - E(R_i)] \times [R_j - E(R_j)]\}$$

Correlation

Measures the strength of the linear relationship between two random variables. It ranges from –1 to +1.

$$Corr\left(R_i, R_j\right) = \frac{Cov\left(R_i, R_j\right)}{\sigma(R_i)\sigma\left(R_j\right)}$$

Sums of Random Variables

If *X* and *Y* are any random variables:

$$E(X + Y) = E(X) + E(Y)$$

If *X* and *Y* are independent random variables:

$$Var(X + Y) = Var(X) + Var(Y)$$

If *X* and *Y* are NOT independent:

$$Var(X + Y) = Var(X) + Var(Y) + 2 \times Cov(X,Y)$$

Skewness and Kurtosis

Skewness, or skew, refers to the extent to which a distribution is not symmetrical. The skewness of a normal distribution is equal to zero.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail.

Kurtosis is a measure of the degree to which a distribution is more or less "peaked" than a normal distribution. Excess kurtosis = kurtosis – 3.

- **Leptokurtic** describes a distribution that is more peaked than a normal distribution.
- **Platykurtic** refers to a distribution that is less peaked, or flatter, than a normal distribution.

Desirable Properties of an Estimator

- An *unbiased* estimator is one for which the expected value of the estimator is equal to the parameter you are trying to estimate.
- An unbiased estimator is also *efficient* if the variance of its sampling distribution is smaller than all the other unbiased estimators of the parameter you are trying to estimate.
- A *consistent* estimator is one for which the accuracy of the parameter estimate increases as the sample size increases.
- A point estimate should be a *linear* estimator when it can be used as a linear function of sample data.

Continuous Uniform Distribution

Distribution where the probability of *X* occurring in a possible range is the length of the range relative to the total of all possible values. Letting *a* and *b* be the lower and upper limits of the uniform distribution, respectively, then for $a \le x_1 < x_2 \le b$:

$$P(x_1 \le X \le x_2) = \frac{(x_2 - x_1)}{(b - a)}$$

Binomial Distribution

Evaluates a random variable with two possible outcomes over a series of *n* trials. The probability of "success" on each trial equals:

p(x) = (number of ways to choose x from n) $p^x(1 - p)^{n-x}$

For a binomial random variable:

expected value = np

variance = $np(1 - p)$

Poisson Distribution

Poisson random variable *X* refers to the number of successes per unit. The parameter lambda (λ) refers to the average number of successes per unit. For the distribution, both its mean and variance are equal to the parameter, λ.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Normal Distribution

Normal distribution is completely described by its mean and variance.

- 68% of observations fall within ± 1s.
- 90% of observations fall within ± 1.65s.
- 95% of observations fall within ± 1.96s.
- 99% of observations fall within ± 2.58s.

Standardized Random Variables

A *standardized random variable* is normalized so that it has a mean of zero and a standard deviation of 1.

z-score: represents number of standard deviations a given observation is from a population mean.

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Central Limit Theorem

When selecting simple random samples of size *n* from a population with mean μ and finite variance σ^2, the sampling distribution of sample means approaches the normal probability distribution with mean μ and variance equal to σ^2/n as the sample size becomes large.

Population and Sample Mean

The **population mean** sums all observed values in the population and divides by the number of observations in the population, *N*.

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

The **sample mean** is the sum of all values in a sample of a population, ΣX, divided by the number of observations in the sample, *n*. It is used to make *inferences* about the population mean.

Population and Sample Variance

The **population variance** is defined as the average of the squared deviations from the mean. The population **standard deviation** is the square root of the population variance.

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

The **sample variance**, s^2, is the measure of dispersion that applies when we are evaluating a sample of *n* observations from a population. Using n – 1 instead of *n* in the denominator improves the statistical properties of s^2 as an estimator of σ^2.

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}$$

Sample Covariance

$$\text{covariance} = \sum_{i=1}^{n} \frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

Standard Error

The **standard error of the sample mean** is the standard deviation of the distribution of the sample means. When the standard deviation of the population, σ, is *known*, the standard error of the sample mean is calculated as:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Confidence Interval

If the population has a *normal distribution with a known variance*, a confidence interval for the population mean is:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$z_{\alpha/2}$ = 1.65 for 90% confidence intervals (significance level 10%, 5% in each tail)

$z_{\alpha/2}$ = 1.96 for 95% confidence intervals (significance level 5%, 2.5% in each tail)

$z_{\alpha/2}$ = 2.58 for 99% confidence intervals (significance level 1%, 0.5% in each tail)

Hypothesis Testing

Null hypothesis (H_0): hypothesis the researcher wants to reject; hypothesis that is actually tested; the basis for selection of the test statistics.

Alternative hypothesis (H_A): what is concluded if there is significant evidence to reject the null hypothesis.

One-tailed test: tests whether value is greater than or less than another value. For example:

H_0: $\mu \le 0$ versus H_A: $\mu > 0$

Two-tailed test: tests whether value is different from another value. For example:

H_0: $\mu = 0$ versus H_A: $\mu \neq 0$

T-Distribution

The *t-distribution* is a bell-shaped probability distribution that is symmetrical about its mean. It is the appropriate distribution to use when constructing confidence intervals based on small samples from populations with unknown *variance* and a normal, or approximately normal, distribution.

$$\text{t-test: } t = \frac{x - \mu}{s / \sqrt{n}}$$

Chi-Square Distribution

The *chi-square test* is used for hypothesis tests concerning the variance of a normally distributed population.

$$\text{chi-square test: } \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

F-Distribution

The *F-test* is used for hypotheses tests concerning the equality of the variances of two populations.

$$\text{F-test: } F = \frac{s_1^2}{s_2^2}$$

Simple Linear Regression

$Y_i = B_0 + B_1 \times X_i + \varepsilon_i$

where:

Y_i = dependent or explained variable

X_i = independent or explanatory variable

B_0 = intercept coefficient

B_1 = slope coefficient

ε_i = error term

Total Sum of Squares

For the dependent variable in a regression model, there is a total sum of squares (TSS) around the sample mean.

total sum of squares = explained sum of squares + sum of squared residuals

TSS = ESS + SSR

$$\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y} - \bar{Y})^2 + \sum (Y_i - \hat{Y})^2$$

Coefficient of Determination

Represented by R^2, it is a measure of the "goodness of fit" of the regression.

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{SSR}{TSS}$$

In a simple two-variable regression, the square root of R^2 is the correlation coefficient (r) between X_i and Y_i. If the relationship is positive, then:

$$r = \sqrt{R^2}$$

Standard Error of the Regression (SER)

Measures the degree of variability of the actual *Y*-values relative to the estimated *Y*-values from a regression equation. The SER gauges the "fit" of the regression line. The smaller the standard error, the better the fit.

Linear Regression Assumptions

- A linear relationship exists between the dependent and the independent variable.
- The independent variable is uncorrelated with the error terms.
- The expected value of the error term is zero.
- The variance of the error term is constant for all independent variables.
- No serial correlation of the error terms.
- The model is correctly specified (does not omit variables).

Regression Assumption Violations

Heteroskedasticity occurs when the variance of the residuals is not the same across all observations in the sample.

Multicollinearity refers to the condition when two or more of the independent variables, or linear combinations of the independent variables, in a multiple regression are highly correlated with each other.

Serial correlation refers to the situation in which the residual terms are correlated with one another.

Multiple Linear Regression

A *simple regression* is the two-variable regression with one dependent variable, Y_i, and one independent variable, X_i. A *multivariate regression* has more than one independent variable.

$$Y_i = B_0 + B_1 \times X_{1i} + B_2 \times X_{2i} + \varepsilon_i$$

Adjusted R-Squared

Adjusted R^2 is used to analyze the importance of an added independent variable to a regression.

$$\text{adjusted } R^2 = 1 - (1 - R^2) \times \frac{n-1}{n-k-1}$$

The F-Statistic

The *F-stat* is used to test whether at least one of the independent variables explains a significant portion of the variation of the dependent variable.

The *homoskedasticity-only F-stat* can only be derived from R^2 when the error terms display homoskedasticity.

Forecasting Model Selection

Model selection criteria takes the form of *penalty factor times mean squared error (MSE)*.

MSE is computed as:

$$\sum_{t=1}^{T} e_t^2 / T$$

Penalty factors for unbiased MSE (s^2), Akaike information criterion (AIC), and Schwarz information criterion (SIC) are: $(T / T - k)$, $e^{(2k/T)}$, and $T^{(k/T)}$, respectively.

SIC has the largest penalty factor and is the most consistent selection criteria.

Covariance Stationary

A time series is covariance stationary if its mean, variance, and covariances with lagged and leading values do not change over time. Covariance stationarity is a requirement for using autoregressive (AR) models. Models that lack covariance stationarity are unstable and do not lend themselves to meaningful forecasting.

Autoregressive (AR) Process

The first-order autoregressive process [AR(1)] is specified as a variable regressed against itself in lagged form. It has a mean of zero and a constant variance.

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

EWMA Model

The exponentially weighted moving average (EWMA) model assumes weights decline exponentially back through time. This assumption results in a specific relationship for variance in the model:

$$\sigma_n^2 = (1 - \lambda) r_{n-1}^2 + \lambda \sigma_{n-1}^2$$

where:

λ = weight on previous volatility estimate (between zero and one)

High values of λ will minimize the effect of daily percentage returns, whereas low values of λ will tend to increase the effect of daily percentage returns on the current volatility estimate.

GARCH Model

A GARCH(1,1) model incorporates the most recent estimates of variance and squared return, and also includes a variable that accounts for a long-run average level of variance.

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2$$

where:

α = weighting on previous period's return

β = weighting on previous volatility estimate

ω = weighted long-run variance

V_L = long-run average variance $= \dfrac{\omega}{1 - \alpha - \beta}$

$\alpha + \beta < 1$ for stability

The EWMA is nothing other than a special case of a GARCH(1,1) volatility process, with $\omega = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$.

The sum $\alpha + \beta$ is called the persistence, and if the model is to be stationary over time (with reversion to the mean), the sum must be less than one.

Simulation Methods

Monte Carlo simulations can model complex problems or estimate variables when there are small sample sizes. Basic steps are: (1) specify data generating process, (2) estimate unknown variable, (3) save estimate from step 2, and (4) go back to step 1 and repeat process *N* times.

Bootstrapping simulations repeatedly draw data from historical data sets and replace data so it can be re-drawn. Requires no assumptions with respect to the true distribution of parameter estimates. However, it is ineffective when there are outliers or when data is non-independent.

FINANCIAL MARKETS AND PRODUCTS

Option and Forward Contract Payoffs

The payoff on a call option to the option buyer is calculated as follows: $C_T = \max(0, S_T - X)$

The price paid for the call option, C_0, is referred to as the call premium. Thus, the profit to the option buyer is calculated as follows:

profit = $C_T - C_0$

The payoff on a put option is calculated as follows:

$P_T = \max(0, X - S_T)$

The payoff to a long position in a forward contract is calculated as follows:

payoff = $S_T - K$

where:

S_T = spot price at maturity

K = delivery price

Futures Market Participants

Hedgers: lock-in a fixed price in advance.

Speculators: accept the price risk that hedgers are unwilling to bear.

Arbitrageurs: interested in market inefficiencies to obtain riskless profit.

Basis

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.

Minimum Variance Hedge Ratio

The *hedge ratio* minimizes the variance of the combined hedge position. This is also the beta of spot prices with respect to futures contract prices.

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

Hedging With Stock Index Futures

$$\text{\# of contracts} = \beta_P \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

Adjusting Portfolio Beta

If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta.

$$\text{\# of contracts} = (\text{target beta} - \text{portfolio beta})\frac{\text{portfolio value}}{\text{underlying asset}}$$

Forward Interest Rates

Forward rates are interest rates implied by the spot curve for a specified future period. The forward rate between T_1 and T_2 can be calculated as:

$$R_{forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left(\frac{T_1}{T_2 - T_1}\right)$$

Forward Rate Agreement (FRA) Cash Flows

An FRA is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$

$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:
L = principal
R_K = annualized rate on L
R = annualized actual rate
T_i = time *i* expressed in years

Cost-of-Carry Model

Forward price when underlying asset does not have cash flows:

$$F_0 = S_0e^{rT}$$

Forward price when underlying asset has cash flows, *I*:

$$F_0 = (S_0 - I)e^{rT}$$

Forward price with continuous dividend yield, *q*:

$$F_0 = S_0e^{(r-q)T}$$

Forward price with storage costs, *u*:

$$F_0 = (S_0 + U)e^{rT} \text{ or } F_0 = S_0e^{(r+u)T}$$

Forward price with convenience yield, *c*:

$$F_0 = S_0e^{(r-c)T}$$

Forward foreign exchange rate using interest rate parity (IRP):

$$F_0 = S_0e^{(r_d - r_f)T}$$

Arbitrage: Remember to buy low, sell high.

- If $F_0 > S_0e^{rT}$, borrow, buy spot, sell forward today; deliver asset, repay loan at end.
- If $F_0 < S_0e^{rT}$, short spot, invest, buy forward today; collect loan, buy asset under futures contract, deliver to cover short sale.

Backwardation and Contango

- Backwardation refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset.
- Contango refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.

Treasury Bond Futures

In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

$$\text{cash received by the short} = (QFP \times CF) + AI$$
$$\text{cost to purchase bond} = QBP + AI$$

where:
QFP = quoted futures price
CF = conversion factor
QBP = quoted bond price
AI = accrued interest

The CTD is the bond that minimizes the following: QBP – (QFP × CF). This formula calculates the cost of delivering the bond.

Duration-Based Hedge Ratio

The objective of a *duration-based hedge* is to create a combined position that does not change in value when yields change by a small amount.

$$\text{\# of contracts} = -\frac{\text{portfolio value} \times \text{duration}_P}{\text{futures value} \times \text{duration}_F}$$

Interest Rate Swaps

Plain vanilla interest rate swap: exchanges fixed for floating-rate payments over the life of the swap. At inception, the value of the swap is zero. After inception, the value of the swap is the difference between the present value of the remaining fixed- and floating-rate payments:

$$V_{\text{swap to pay fixed}} = B_{float} - B_{fix}$$
$$V_{\text{swap to receive fixed}} = B_{fix} - B_{float}$$
$$B_{fixed} = (PMT_{fixed,t_1} \times e^{-rt_1}) + (PMT_{fixed,t_2} \times e^{-rt_2}) + \ldots + [(\text{notional} + PMT_{fixed,t_n}) \times e^{-rt_n}]$$
$$B_{floating} = \left[\text{notional} + \left(\text{notional} \times r_{float}\right)\right] \times e^{-rt_1}$$

Currency Swaps

Exchanges payments in two different currencies; payments can be fixed or floating. If a swap has a positive value to one counterparty, that party is exposed to credit risk.

$$V_{swap}(DC) = B_{DC} - (S_0 \times B_{FC})$$

where:
S_0 = spot rate in DC per FC

Option Pricing Bounds

Upper bound European/American call:

$c \le S_0$; $C \le S_0$

Upper bound European/American put:

$p \le Xe^{-rT}$; $P \le X$

Lower bound European call on non-dividend-paying stock:

$$c \ge \max(S_0 - Xe^{-rT}, 0)$$

Lower bound European put on non-dividend-paying stock:

$$p \ge \max(Xe^{-rT} - S_0, 0)$$

Exercising American Options

- It is never optimal to exercise an American call on a non-dividend-paying stock before its expiration date.
- American puts can be optimally exercised early if they are sufficiently in-the-money.
- An American call on a dividend-paying stock may be exercised early if the dividend exceeds the amount of forgone interest.

Put-Call Parity

$$p = c - S + Xe^{-rT}$$
$$c = p + S - Xe^{-rT}$$

Covered Call and Protective Put

Covered call: Long stock plus short call.
Protective put: Long stock plus long put. Also called portfolio insurance.

Option Spread Strategies

Bull spread: Purchase call option with low exercise price and subsidize the purchase with sale of a call option with a higher exercise price.
Bear spread: Purchase call with high strike price and short call with low strike price.
Investor keeps difference in price of the options if stock price falls. Bear spread with puts involves buying put with high exercise price and selling put with low exercise price.
Butterfly spread: Three different options: buy one call with low exercise price, buy another with a high exercise price, and short two calls with an exercise price in between. Butterfly buyer is betting the stock price will stay near the price of the written calls.
Calendar spread: Two options with different expirations. Sell a short-dated option and buy a long-dated option. Investor profits if stock price stays in a narrow range.
Diagonal spread: Similar to a calendar spread except that the options can have different strike prices in addition to different expirations.
Box spread: Combination of bull call spread and bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price minus the low exercise price.

Option Combination Strategies

Long straddle: Bet on volatility. Buy a call and a put with the same exercise price and expiration date. Profit is earned if stock price has a large change in either direction.
Short straddle: Sell a put and a call with the same exercise price and expiration date. If stock price remains unchanged, seller keeps option premiums. Unlimited potential losses.
Strangle: Similar to straddle except purchased option is out-of-the-money, so it is cheaper to implement. Stock price has to move more to be profitable.

Strips and straps: Add an additional put (strip) or call (strap) to a straddle strategy.

Exotic Options

Gap option: payoff is increased or decreased by the difference between two strike prices.
Compound option: option on another option.
Chooser option: owner chooses whether option is a call or a put after initiation.
Barrier option: payoff and existence depend on price reaching a certain barrier level.
Binary option: pay either nothing or a fixed amount.
Lookback option: payoff depends on the maximum (call) or minimum (put) value of the underlying asset over the life of the option. This can be fixed or floating depending on the specification of a strike price.
Shout option: owner receives intrinsic value of option at shout date or expiration, whichever is greater.
Asian option: payoff depends on average of the underlying asset price over the life of the option; less volatile than standard option.
Basket options: options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as *rainbow options*.

Foreign Currency Risk

A net long (short) currency position means a bank faces the risk that the FX rate will fall (rise) versus the domestic currency.

net currency exposure = (assets − liabilities) + (bought − sold)

On-balance sheet hedging: matched maturity and currency foreign asset-liability book.
Off-balance sheet hedging: enter into a position in a forward contract.

Central Counterparties (CCPs)

When trades are centrally cleared, a CCP becomes the seller to a buyer and the buyer to a seller.
Advantages of CCPs: transparency, offsetting, loss mutualization, legal and operational efficiency, liquidity, and default management.
Disadvantages of CCPs: moral hazard, adverse selection, separation of cleared and non-cleared products, and margin procyclicality.
Risks faced by CCPs: default risk, model risk, liquidity risk, operational risk, and legal risk. Default of a clearing member and its flow through effects is the most significant risk for a CCP.

MBS Prepayment Risk

Factors that affect prepayments:

- Prevailing mortgage rates, including (1) spread of current versus original mortgage rates, (2) mortgage rate path (refinancing burnout), and (3) level of mortgage rates.
- Underlying mortgage characteristics.
- Seasonal factors.
- General economic activity.

Conditional Prepayment Rate (CPR)

Annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. The single monthly mortality (SMM) rate is derived from CPR and used to estimate monthly prepayments for a mortgage pool:

$$SMM = 1 - (1 - CPR)^{1/12}$$

Option-Adjusted Spread (OAS)

- Spread after the "optionality" of the cash flows is taken into account.
- Expresses the difference between price and theoretical value.
- When comparing two MBSs of similar credit quality, buy the bond with the biggest OAS.
- OAS = zero-volatility spread − option cost.

VALUATION AND RISK MODELS

Value at Risk (VaR)

Minimum amount one could expect to lose with a given probability over a specific period of time.

$$VaR(X\%) = z_{X\%} \times \sigma$$

Use the square root of time to change daily to monthly or annual VaR.

$$VaR(X\%)_{J\text{-days}} = VaR(X\%)_{1\text{-day}} \sqrt{J}$$

VaR Methods

The ***delta-normal method*** (a.k.a. the variance-covariance method) for estimating VaR requires the assumption of a normal distribution. The method utilizes the expected return and standard deviation of returns.
The ***historical simulation method*** for estimating VaR uses historical data. For example, to calculate the 5% daily VaR, you accumulate a number of past daily returns, rank the returns from highest to lowest, and then identify the lowest 5% of returns.
The ***Monte Carlo simulation method*** refers to computer software that generates many possible outcomes from the distributions of inputs specified by the user. All of the examined portfolio returns will form a distribution, which will approximate the normal distribution. VaR is then calculated in the same way as with the delta-normal method.

Expected Shortfall (ES)

- Average or expected value of all losses greater than the VaR: $E[L_p \mid L_p > VaR]$.
- Popular measure to report along with VaR.
- ES is also known as conditional VaR or expected tail loss.
- Unlike VaR, ES has the ability to satisfy the coherent risk measure property of subadditivity.

Binomial Option Pricing Model

A *one-step binomial model* is best described within a two-state world where the price of a stock will either go up once or down once, and the change will occur one step ahead at the end of the holding period.
In the *two-period binomial model* and multi-period models, the tree is expanded to provide for a greater number of potential outcomes.

Step 1: Calculate option payoffs at the end of all states.
Step 2: Calculate option values using risk-neutral probabilities.

$$\text{size of up move} = U = e^{\sigma\sqrt{T}}$$

$$\text{size of down move} = D = \frac{1}{U}$$

$$\pi_{up} = \frac{e^{rt} - D}{U - D};\ \pi_{down} = 1 - \pi_{up}$$

Step 3: Discount to today using risk-free rate.

π_{up} can be altered so that the binomial model can price options on stocks with dividends, stock indices, currencies, and futures.
Stocks with dividends and stock indices: replace e^{rT} with $e^{(r-q)T}$, where q is the dividend yield of a stock or stock index.
Currencies: replace e^{rT} with $e^{(r-r_f)T}$, where r_f is the foreign risk-free rate of interest.
Futures: replace e^{rT} with 1 since futures are considered zero growth instruments.

Black-Scholes-Merton Model

$$c = S_0 \times N(d_1) - Xe^{-rT}N(d_2)$$
$$p = Xe^{-rT}N(-d_2) - S_0N(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[r + 0.5 \times \sigma^2\right] \times T}{\sigma \times \sqrt{T}}$$

$d_2 = d_1 - \left(\sigma \times \sqrt{T}\right)$
T = time to maturity
S_0 = asset price
X = exercise price
r = risk-free rate
σ = stock return volatility
$N(\bullet)$ = cumulative normal probability

Greeks

Delta: estimates the change in value for an option for a one-unit change in stock price.

- Call delta between 0 and +1; increases as stock price increases.
- Call delta close to 0 for far out-of-the-money calls; close to 1 for deep in-the-money calls.
- Put delta between −1 and 0; increases from −1 to 0 as stock price increases.
- Put delta close to 0 for far out-of-the-money puts; close to −1 for deep in-the-money puts.
- The delta of a forward contract is equal to 1.
- The delta of a futures contract is equal to e^{rT}.
- When the underlying asset pays a dividend, q, the delta must be adjusted. If a dividend yield exists, delta of call equals $e^{-qT} \times N(d_1)$, delta of put equals $e^{-qT} \times [N(d_1) - 1]$, delta of forward equals e^{-qT}, and delta of futures equals $e^{(r-q)T}$.

Theta: time decay; change in value of an option for a one-unit change in time; more negative when option is at-the-money and close to expiration.
Gamma: rate of change in delta as underlying stock price changes; largest when option is at-the-money.
Vega: change in value of an option for a one-unit change in volatility; largest when option is at-the-money; close to 0 when option is deep in- or out-of-the-money.
Rho: sensitivity of option's price to changes in the risk-free rate; largest for in-the-money options.

Delta-Neutral Hedging

- To completely hedge a long stock/short call position, purchase shares of stock equal to delta × number of options sold.
- Only appropriate for small changes in the value of the underlying asset.
- Gamma can correct hedging error by protecting against large movements in asset price.
- Gamma-neutral positions are created by matching portfolio gamma with an offsetting option position.

Bond Valuation

There are three steps in the bond valuation process:
Step 1: Estimate the cash flows. For a bond, there

are two types of cash flows: (1) the annual or semiannual coupon payments and (2) the recovery of principal at maturity, or when the bond is retired.

Step 2: Determine the appropriate discount rate. The approximate discount rate can be either the bond's yield to maturity (YTM) or a series of spot rates.

Step 3: Calculate the PV of the estimated cash flows. The PV is determined by discounting the bond's cash flow stream by the appropriate discount rate(s).

Clean and Dirty Bond Prices

When a bond is purchased, the buyer must pay any accrued interest (AI) earned through the settlement date.

$$\text{AI} = \text{coupon} \times \left(\frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}} \right)$$

Clean price: bond price without accrued interest.
Dirty price: includes accrued interest; price the seller of the bond must be paid to give up ownership.

Compounding

Discrete compounding:

$$FV_n = PV_0\left(1 + \frac{r}{m}\right)^{m \times n}$$

where:
r = annual rate
m = compounding periods per year
n = years

Continuous compounding:

$$FV_n = PV_0 e^{r \times n}$$

Spot Rates

A *t*-period spot rate, denoted as *z(t)*, is the yield to maturity on a zero-coupon bond that matures in *t*-years. It can be calculated using a financial calculator or by using the following formula (assuming periods are semiannual), where *d(t)* is a discount factor:

$$z(t) = 2\left[\left(\frac{1}{d(t)}\right)^{1/2t} - 1\right]$$

Forward Rates

Forward rates are interest rates that span future periods.

$$(1 + \text{forward rate})^t = \frac{(1 + \text{periodic yield})^{t+1}}{(1 + \text{periodic yield})^t}$$

Realized Return

The gross realized return for a bond is its end-of-period total value minus its beginning-of-period value divided by its beginning-of-period value.

$$R_{t-1,t} = \frac{BV_t + C_t - BV_{t-1}}{BV_{t-1}}$$

The net realized return for a bond is its gross realized return minus per period financing costs.

Yield to Maturity (YTM)

The YTM of a fixed-income security is equivalent to its internal rate of return. The YTM is the discount rate that equates the present value of all cash flows associated with the instrument to its price. The yield to maturity assumes cash flows will be reinvested at the YTM and assumes that the bond will be held until maturity.

Relationship Among Coupon, YTM, and Price

If coupon rate > YTM, bond price will be greater than par value: *premium bond*.
If coupon rate < YTM, bond price will be less than par value: *discount bond*.
If coupon rate = YTM, bond price will be equal to par value: *par bond*.

Dollar Value of a Basis Point

The DV01 is the change in a fixed income security's value for every one basis point change in interest rates.

$$DV01 = -\frac{\Delta BV}{10{,}000 \times \Delta y}$$

$$DV01 = \text{duration} \times 0.0001 \times \text{bond value}$$

Effective Duration and Convexity

Duration: first derivative of the price-yield relationship; most widely used measure of bond price volatility; the longer (shorter) the duration, the more (less) sensitive the bond's price is to changes in interest rates; can be used for linear estimates of bond price changes.

$$\text{effective duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

Convexity: measure of the degree of curvature (second derivative) of the price/yield relationship; accounts for error in price change estimates from duration. Positive convexity always has a favorable impact on bond price.

$$\text{convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$$

Bond Price Changes With Duration and Convexity

percentage bond price change ≈ duration effect + convexity effect

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y + \frac{1}{2} \times \text{convexity} \times \Delta y^2$$

Bonds With Embedded Options

Callable bond: issuer has the right to buy back the bond in the future at a set price; as yields fall, bond is likely to be called; prices will rise at a *decreasing rate—negative convexity*.
Putable bond: bondholder has the right to sell bond back to the issuer at a set price.

Country Risk

Sources of country risk: (1) where the country is in the economic growth life cycle, (2) political risks, (3) the legal systems of a country, including both the structure and the efficiency of legal systems, and (4) the disproportionate reliance of a country on one commodity or service.
Factors influencing sovereign default risk: (1) a country's level of indebtedness, (2) obligations such as pension and social service commitments, (3) a country's level of and stability of tax receipts, (4) political risks, and (5) backing from other countries or entities.

Internal Credit Ratings

At-the-point approach: goal is to predict the credit quality over a relatively short horizon of a few months or, more generally, a year.
Through-the-cycle approach: focuses on a longer time horizon and includes the effects of forecasted cycles.

Expected Loss

The *expected loss* (EL) represents the decrease in value of an asset (portfolio) with a given exposure subject to a positive probability of default.

expected loss = exposure amount (EA) × loss rate (LR) × probability of default (PD)

Unexpected Loss

Unexpected loss represents the variability of potential losses and can be modeled using the definition of standard deviation.

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

Operational Risk

Operational risk is defined as: *The risk of direct and indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events.*

Operational Risk Capital Requirements

- *Basic indicator approach*: capital charge measured on a firmwide basis as a percentage of annual gross income.
- *Standardized approach*: banks divide activities among business lines; capital charge = sum for each business line. Capital for each business line determined with beta factors and annual gross income.
- *Advanced measurement approach*: banks use their own methodologies for assessing operational risk. Capital allocation is based on the bank's operational VaR.

Loss Frequency and Loss Severity

Operational risk losses are classified along two independent dimensions:
Loss frequency: the number of losses over a specific time period (typically one year). Often modeled with the *Poisson distribution* (a distribution that models random events).
Loss severity: value of financial loss suffered. Often modeled with the *lognormal distribution* (distribution is asymmetrical and has fat tails).

Stress Testing

VaR tells the probability of exceeding a given loss but fails to incorporate the possible amount of a loss that results from an extreme amount.
Stress testing complements VaR by providing information about the magnitude of losses that may occur in extreme market conditions.

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